Non-Linearity and Non-Gaussianity in Atmospheric Dynamics

Jhon Edinson Hinestroza Ramírez
PhD Student in Mathematical Engineering

PhD. Olga L. Quintero
Advisor

PhD. Angela María Rendón
Co-Advisor

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Motivation

Figure 1: Taken from: *eltiempo.com*

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Motivation

Figure 2: Taken from *eltiempo.com*

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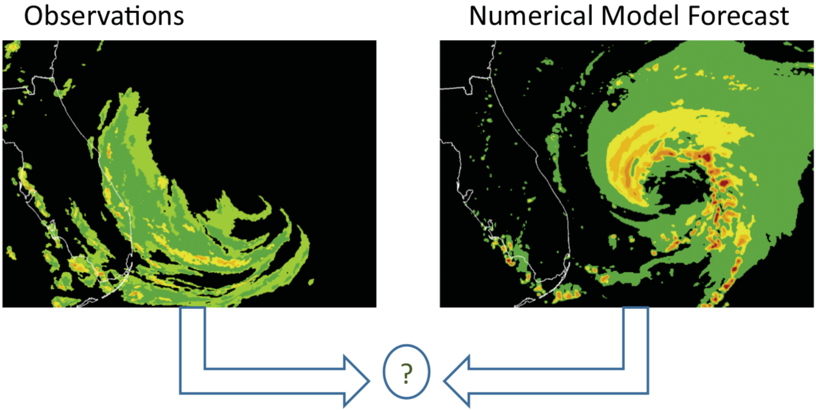
Motivation

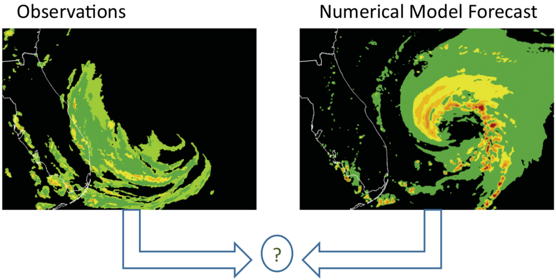


Motivation

Air Pollution

$\Downarrow$

Mathematical models: Data assimilation, particle filters (non-linear particle filters).



Motivation

Mathematical models

(Misenis & Zhang 2010, Tuccella et al. 2012, Carvalho et al. 2012, Hu et al. 2013, Dillon et al. 2016).

Motivation

Mathematical models

$\Downarrow$

- Sensitivity.
- Uncertainty sources.

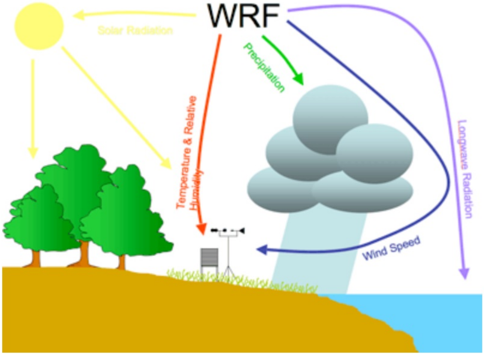
(Misenis & Zhang 2010, Tuccella et al. 2012, Carvalho et al. 2012, Hu et al. 2013, Dillon et al. 2016).

Research Problem

To identify, measure, and model significant sources of uncertainty in the short-term meteorological forecast with the Weather Research and Forecasting (WRF) numerical model and to develop a methodology based on non-linear particle filters for reducing it.

Tools: WRF Model

- The WRF model: numerical weather prediction and atmospheric simulation system.
- The WRF model is used for studying the air quality.



Taken from: Climate Information: Responding to User Needs. University of Maryland. http://www.climateneeds.umd.edu/chesapeake/wrf.html

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(Skamarock et al. 2008, Misenis & Zhang 2010, Carvalho et al. 2012, Tuccella et al. 2012, Hu et al. 2013, Kumar et al. 2016, Dillon et al. 2016).

WRF: Mathematical Models

$$\partial_t U + m_x \left[\partial_x \left(U_u\right) + \cdots\right] = F_U,$$

$$\partial_t V + m_y \left[\partial_x \left(U_v\right) + \cdots\right] = F_V,$$

$$\partial_t W + \left(\frac{m_x m_y}{m_y}\right) \left[\partial_x \left(Uw\right) + \cdots\right] = F_W.$$

Some Problems

- How does accurate solution close to reality?
- How does change the solution of the system when we change these conditions?
- How sensitive is the model for the small changes made to it?
- What is the maximum change in the conditions such that there is no change in the solution?

Sensitivity Analysis (SA)

- The uncertainty (variability) associated with a sensitive parameter in the model.

Sensitivity Analysis (SA)

- The sensitivity analysis tries to response questions in relation to how the variation in the output can associate with variations in the different input factors.
- The sensitivity analysis is a function between the model inputs and the model outputs.

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(Pianosi et al. 2016, Borgonovo & Plischke 2016).

Uncertainty Analysis and Quantification

The uncertainty analysis discriminate the quantify of uncertainty in the output of a model and it is used for uncertainty assessment of numerical models.

(Uusitalo et al. 2015, Pianosi et al. 2016).

Uncertainty Analysis and Quantification

The uncertainty quantification is the science of quantitative characterization and reduction of uncertainties in both computational and real-world applications. It tries to determine how likely certain outcomes are if some aspects of the system are not exactly known.

Uncertainty Analysis and Quantification

Uncertainty, in models of physical systems, is almost always represented as a probability density function (PDF) through samples, parameters, or kernels (objective of uncertainty quantification).

Uncertainty Analysis and Quantification: Steps

1. Define the system of interest, its response, and the desired performance measures.
2. Write a mathematical formulation of the system-governing equations, geometry, and parameter values.

Uncertainty Analysis and Quantification: Steps

3. Formulate a discretized representation and the numerical methods and algorithms for its solution.
4. Perform the simulations and the analysis.
5. Loop back to step 1.

Solution Tools

$$\boxed{\text{Uncertainty Quantification}}$$

$$\Downarrow$$

$$\boxed{\text{Data Assimilation}}$$

Solution Tools

$$\boxed{\text{Uncertainty Quantification}}$$

$$\Downarrow$$

$$\boxed{\text{Data Assimilation}}$$

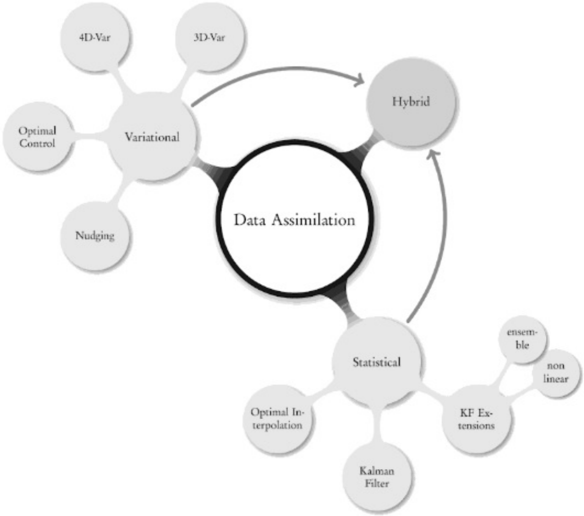
1. Data Assimilation.
2. Particle Filters.

Tool: Data Assimilation

- It is an approach/method for combining observations with model output with the objective of improving the latter.
- Data assimilation combines past knowledge of a system in the form of a numerical model with new information about that system in the form of observations of that system.

(van Leeuwen et al. 2015, Asch et al. 2016).

Data Assimilation: Methods



(Asch et al. 2016).

Particle Filters

- The particle filters can be used to estimate the state of a system. To estimate x, using y.
- The aim: To approximate the relevant probability distributions, with discrete aleatory measures (or continuous) called particles and his weight associates.

(Quintero 2010).

Filtering Problem

Observations $\{Y_t\} \Longrightarrow$ Predictions of State $\{X_t\}$

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Observations $\{Y_t\} \Longrightarrow$ Predictions of State $\{X_t\}$

State X

$$\frac{X_t}{dt} = b(t, X_t) + \sigma(t, X_t)W_t; \;\; t \geq 0,$$

where $b : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^n$, $\sigma : \mathbb{R}^{n+1} \rightarrow \mathbb{R}^{n \times p}$, and W_t is p-dimensional white noise.

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Remark: The aim in a filtering problem: To determine the conditional distribution of X using Y.

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Foundation of the Problem

$$p(x|y) = \frac{p(y|x) \cdot p(x)}{p(y)}$$

How to Approximate the States of the System?

$\boxed{\text{Particle Filters}}$

$\boxed{\text{Particle Filters}}$

$$\Downarrow$$

- The state equations are linear and its PDF "a posteriori" is Gaussian $\rightarrow$ Kalman filter.

How to Approximate the States of the System?

$$\boxed{\text{Particle Filters}}$$

$$\Downarrow$$

- The state equations are non-linear and its PDF "a posteriori" is Gaussian $\rightarrow$ Extended Kalman Filter (EKF).

How to Approximate the States of the System?

$\boxed{\text{Particle Filters}}$

$\Downarrow$

- If the state equations are too much non-linear and its PDF "a posteriori" is no-Gaussian $\rightarrow$ the EKF is not a good solution.

How to Approximate the States of the System?

$\boxed{\text{Non-linear Particle Filters}}$

With a non-linear particle filters is seeked an equation for the conditional PDF of a process unobserved for a trajectory looked $\rightarrow$ Kushner and Zakai equation.

Particle Filters

With a non-linear and no-Gaussian filter the PDF and non-linear functions are approximated to find the solution.

(van Leeuwen et al. 2015).

Particle Filters

- The Gaussian Sum Filter (GSM).
- The Gibbs sampler.
- The Numerical Integration Filter (NIF).
- Montecarlo integration with importance sampling.
- Rejection Sampling Filter (RSF).

(van Leeuwen et al. 2015).

Non-linear Particle Filters

$$\boxed{\text{Non-linear Particle Filters}}$$

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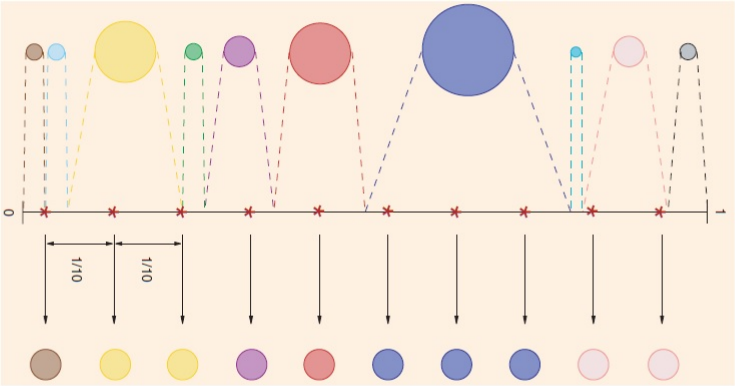
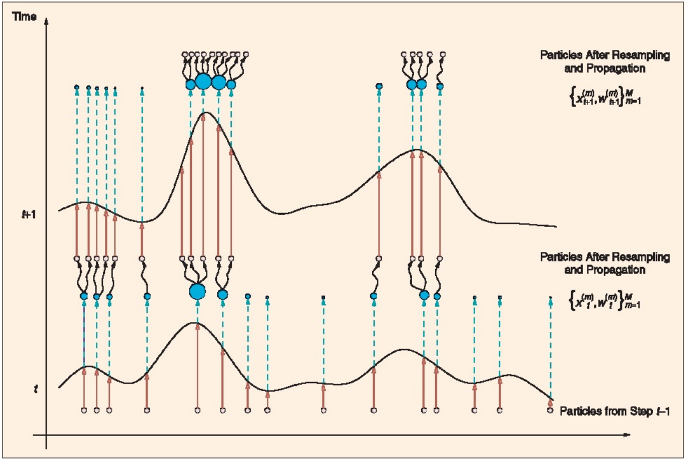
$$\boxed{\text{Approximate Solutions}} \Leftarrow \boxed{\text{Importance Function}}$$

Ressampling



(Djuric et al. 2003).

Description of the Particle Filtering



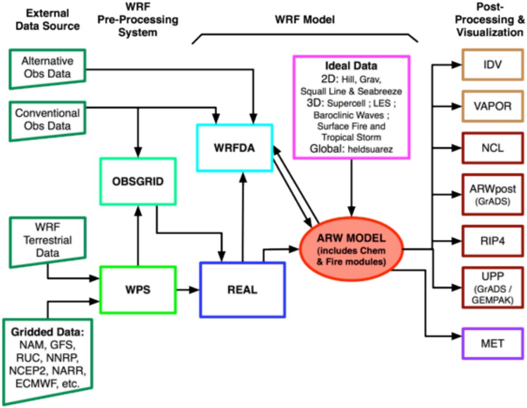
(Djuric et al. 2003).

Initial Importance Function

Some questions appear:

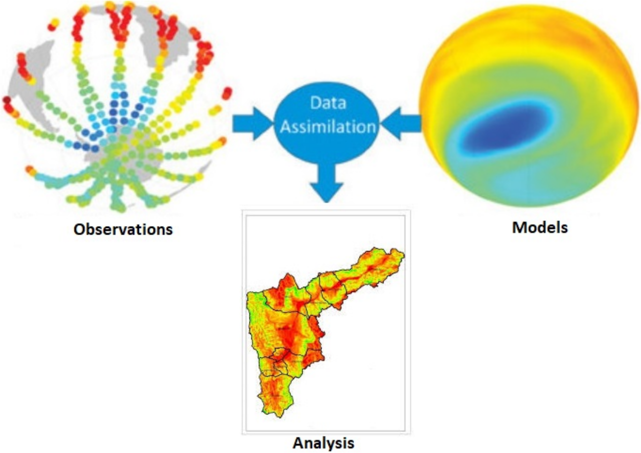
- What initial importance function to use for finding the solution?
- How the solution change with the an initial importance function selected?
- How much the solution change with an initial importance function selected?
- Is there sensitivity of the solution to the initial importance function selected?

Learning about WRF:



Taken from: http://www2.mmm.ucar.edu/wrf/users/docs/user_guide_V4/WRFUsersGuide.pdf Consulted: September-24-2018.

Learning about data assimilation.



Goals

- How does accurate solution close to reality?
- How does change the solution of the system when we change these conditions?
- How sensitive is the model for the small changes made to it?
- What is the maximum change in the conditions such that there is no change in the solution?

Expected Results

- To understand the sensitivity of the WRF model in the Aburrá Valley.
- To identify and reduce its uncertain sources, such that can obtain better results in the monitor of the climate and weather forecast.

Expected Results

- The results will must show the implications of the sensitivity of the model for the air quality modeling in the Aburrá Valley.
- This study will must show the importance of the non-linear data assimilation to forecasting weather modeling.

Thanks!

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